5.8 – Applications of Logarithms

Daily Objectives:

1. Take the logarithm of both sides of an exponential equation to solve it.

Example 1: The equation $y = 1.25 + 0.72(0.954)^{x-10}$ gives the greatest distance from a motion sensor for each swing of the pendulum based on the number of the swing. Use this equation to find the swing number when the greatest distance will be closest to 1.47m. Explain each step.

$1.47 = 1.25 + .72(.954)^{x-10}$

$\frac{.22}{.72} = \frac{.72(.954)^{x-10}}{.72}$

$\frac{.22}{.72} = .954^{x-10}$

$\log\left(\frac{.22}{.72}\right) = \log(.954)^{x-10}$

$\frac{\log\left(\frac{.22}{.72}\right)}{\log .954} = \frac{(x-10)(\log .954)}{\log .954}$

$\frac{\log\left(\frac{.22}{.72}\right)}{\log .954} + 10 = x$

$\boxed{x = 35.177}$

Solving with Logarithms

If $a^x = b$, and a and b are both positive, then $\log a^x = \log b$.

Example 2: Solve $5 = 2^x$ by (1) taking the log of both sides and (2) using the properties of logarithms.

$\log 5 = \log 2^x$

$\frac{\log 5}{\log 2} = \frac{x \log 2}{\log 2}$

$\boxed{2.322 = x}$

Prove $(10^a)^b = 10^{ab}$ by taking the log of both sides and using properties of logarithms.

$\log\left[(10^a)^b\right] = \log(10ab)$

$b \log 10^a = ab \log 10$

$ab \log 10 = ab \log 10$

$ab = ab$

$\log_{10} 10 = x$

$10^x = 10$

$x = 1$

Example 3: You let a warm cup of tea set out on the counter and proceed to take the temperature of the tea every 10 seconds. The data you collected is listed below. Use the data to answer the following questions:

X (time)	Y (Celsius)
0	7.7495
10	6.87
20	5.4995
30	4.4995
40	3.812
50	3.312
60	2.8745
70	2.437
80	2.962
90	1.8745
100	1.6245
110	1.3745
120	1.187
130	0.9995
140	0.812
150	0.687
160	0.562
170	0.437
180	0.312

i. Enter the data into your calculator and sketch a graph of the points.

ii. Find the exponential function using ExpReg for the data listed to the left.

$y = 8.0964(.9835)^x$

iii. After how many minutes is the temperature 5 degrees Celsius?

$5 = 8.0964(.9835)^x$

$\frac{5}{8.0964} = .9835^x$

$\log\left(\frac{5}{8.0964}\right) = \log(.9835^x)$

$\log\left(\frac{5}{8.0964}\right) = x \log(.9835)$

$\frac{\log\left(\frac{5}{8.0964}\right)}{\log(.9835)} = x$

$\boxed{28.969s \approx x}$

iv. What is the temperature after 122 seconds?

$y = 8.0964(.9835)^{122}$

$\boxed{y = 1.064°C}$

Example 4: Solve each equation.

a. $9.5(8^x) = 220$

$8^x = \frac{220}{9.5}$

$\log 8^x = \log\left(\frac{220}{9.5}\right)$

$\frac{x \log 8}{\log 8} = \frac{\log\left(\frac{220}{9.5}\right)}{\log 8}$

$\boxed{x \approx 1.511}$

b. $0.405 = 15.6(0.72)^x - 7$

$\frac{7.405}{15.6} = \frac{15.6(.72)^x}{15.6}$

$\log\left(\frac{7.405}{15.6}\right) = \log(.72)^x$

$\frac{\log\left(\frac{7.405}{15.6}\right)}{\log(.72)} = \frac{x \log(.72)}{\log(.72)}$

$\boxed{2.268 \approx x}$

Example 5: Suppose that you invest \$5,000 in a savings account. How long would it take you to double your money with 5% interest compounded annually?

$10{,}000 = 5{,}000(1.05)^x$

$2 = 1.05^x$

$\log 2 = \log(1.05)^x$

$$\frac{\log 2}{\log 1.05} = \frac{x \log 1.05}{\log 1.05}$$

$\boxed{14.207\text{ y} \approx x}$

Example 6: The population of an animal species introduced into an area sometimes increases rapidly at first and then more slowly over time. A logarithmic function models this kind of growth. Suppose that a population of N deer in an area t months after the deer are introduced is given by the equation $N = 325\log(4t+2)$. How long will it take for the deer population to reach 800? [Round to the nearest whole month.]

$800 = 325\log(4t+2)$

$\frac{800}{325} = \log(4t+2)$

$10^{\frac{32}{13}} = 4t+2$

$$\frac{10^{\frac{32}{13}} - 2}{4} = \frac{4t}{4}$$

$71.857 = t$

$\boxed{72 \text{ months}}$